INTEGRABILITY OF TWIST - 3 EVOLUTION EQUATIONS IN QCD

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I review the recent progress in solution of the evolution equations of the three particle hadron distribution amplitudes.

The three particle operators appear in QCD in the studies of the twist − 3 parton distributions [1,2,3] and twist − 3 meson wave functions [4,5], and more interestingly, leading twist nucleon wave functions [6,7,8]. For example, the Q^2 dependence of a baryon wave function is given to one loop accuracy by the following expression

$$\phi(x_i, Q^2) = x_1 x_2 x_3 \sum_{N,q} \phi_{N,q}\, P_{N,q}(x_i) \left(\frac{\alpha_s(Q)}{\alpha_s(Q_0)}\right)^{\gamma_{N,q}/b_0}, \tag{1}$$

where $b_0 = 11/3\, N_c - 2/3\, n_f$. The anomalous dimensions of the three quark operators, $\gamma_{N,q}$, and polynomials, $P_{N,q}(x_i)$ arise from the solution of the eigenvalue problem for a certain integral operator (Hamiltonian) H, which effectively describes the dynamics of the three particle system with a pairwise interaction. It has been observed recently [9] that for the particular twist-3 operators the corresponding three particle systems are intrisically related to the Heisenberg spin magnet, which is known to be integrable quantum mechanical model and their solution can be found by applying powerful method of Integrable Models.

It is well known that the conformal symmetry of the QCD Lagrangian reveals itself as the $SL(2,R)$ invariance of the evolutions equations [10,11]. As a consequence the pairwise Hamiltonians depend on the corresponding two - particle Casimir operators only,

$$H = \sum_{ik} H_{ik}, \quad H_{ik} = h(J_{ik}), \quad J_{ik}(J_{ik}+1) = L_{ik}^2 = (\vec{L}_i + \vec{L}_k)^2, \tag{2}$$

where L_i is the $SL(2,R)$ generators associated with i-th particle. Then the explicit calculations [13] yield that up to unessential constant the Hamiltonian governing the Q^2 evolution of the wave function of baryon with the maximal helicity $\lambda = 3/2$

$$H_{3/2} = 2\left(1 + \frac{1}{N_c}\right) \sum_{i<k} [\psi(J_{ik}+1) - \psi(2)] + \frac{3}{2} C_F \tag{3}$$

coincides with the Hamiltonian of the noncompact $XXX_{s=-1}$ Heisenberg spin magnet. This Hamiltonian possesses an additional integral of motion (conserved charge)

$$Q = \frac{i}{2}[L_{12}^2, L_{23}^2] = i^3 \partial_1 \partial_2 \partial_3 z_{12} z_{23} z_{31}\,, \tag{4}$$

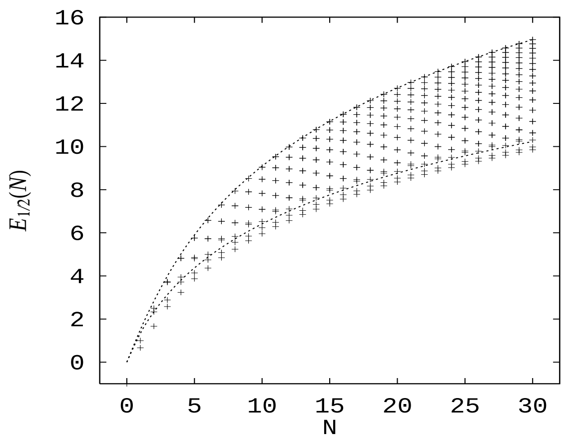

Figure 1: The spectrum of eigenvalues for the Hamiltonian $\mathcal{H}_{12}$. The lines of the largest and the smallest eigenvalues of $\mathcal{H}_{32}$ are indicated by dots for comparison.

that commutes with $H_{3/2}$ and with $SL(2)$ generators. It is interesting to note that this model has already been encountered in QCD in the studies of the Regge asymptotics of the scattering amplitudes [15,16]. The eigenvalue problem for the operator Q appears to be much more simple, though equivalent, those for the $H_{3/2}$ Hamiltonian, and a number of results had been obtained in the Refs. [9,12,17,18,19] for their spectra. In particular, using the different metods it became possible to get very accurate decription of the spectrum of the operator Q, and the Hamiltonian $H_{3/2}$. Moreover the obtained results may serve a starting point to analysis of the more interesting from the physical point of view case of the $\lambda = 1/2$ nucleon distribution amplitudes.

The scale dependence of the $\lambda = 1/2$ nucleon distribution amplitudes is driven by the Hamiltonian

$$H_{1/2} = H_{3/2} + V, \qquad V = -\left(L_{12}^{-2} + L_{23}^{-2}\right). \tag{5}$$

The Hamiltonian $H_{1/2}$ is not integrable and the corresponding eigenproblem cannot be solved exactly. Hovewer, the operator V can be treated as a small perturbation to the Hamiltonian $H_{3/2}$ for the most part of the spectrum, except a few low lying levels. The numerical calculation gives the spectrum shown on Fig. 1. The spectra of $H_{1/2}$ and $H_{3/2}$ are very similar in the upper part and at the same time two lowest levels of the $H_{1/2}$ Hamiltonian appears to be special and 'dive' considerably below the line of the lowest eigenvalue of $H_{3/2}$. The careful analysis of the low energy part of the spectrum* allows to to explain this phenomenon as due to binding of the quarks with opposite helicity and forming the scalar diquark [12].

The above analysis can be extended to the case of the twist - 3 quark gluon operators, which had been paid much attention recently [1,2,3,21]. The Q^2 evolution of the twist-3 quark gluon distribution amplitudes are driven by the Hamiltonians (in the large N_c limit)

$$\begin{aligned} H_S &= \psi(J_{12}+\frac{5}{2}) + \psi(J_{12}-\frac{1}{2}) + \psi(J_{23}+\frac{3}{2}) + \psi(J_{23}+\frac{1}{2}) - \delta, & (6)\\ H_T &= \psi(J_{12}+\frac{5}{2}) + \psi(J_{12}-\frac{1}{2}) + \psi(J_{23}+\frac{5}{2}) + \psi(J_{23}-\frac{1}{2}) - \delta, & (7) \end{aligned}$$

for the chiral even and chiral odd distributions, respectively, and $\delta = 4\psi(2) + 3/2$. The both

* It turns out that the resulting problem for the effective Hamiltonian describing dynamics of the low lying levels turns out to be a generalization of the famous Kroning-Penney model of a single particle in a periodic delta potential [20].

Hamiltonians possess the integrals of motion [9] which are

$$Q_S = \left\{L_{12}^2, L_{23}^2\right\} - \frac{1}{2}L_{23}^2 - \frac{9}{2}L_{12}^2, \tag{8}$$

$$Q_T = \left\{L_{12}^2, L_{23}^2\right\} - \frac{9}{2}L_{23}^2 - \frac{9}{2}L_{12}^2. \tag{9}$$

The underlying models can be identified as the open nonhomogeneous noncompact spin chains. The theory of the open spin chains is less developed as compared with those for the closed spin chains, and many problems has to be solved yet (see Ref. [22]). However many results (such as calculation of the asymptotic expansion for conserved charges and energies) are available by the elementary methods [9,21], the full account can be found in Ref. [21].

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